**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Solution: -

We’ve a normal distribution with µ = 45 and σ = 8 let X be the amount of time it takes to complete the repair on a customer’s car. To finish in one hour you must have X ≤ 50 so the question is to find P(X > 50).

P(X>50) = 1-P(X≤50)

Z(X-50) = (X-45)/8

Thus, the question can be answered by using normal table to find.

P(X≤50) = P(Z≤(50-45)/8)

= P(Z≤0.625)

= 73.4%

Probability that the service manager will not meet his demand will be = 100-73.4

= 26.6%

(or) = 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Solution: -

1. We have a normal distribution µ = 38 and σ = 6. Let be the number of employees, so acc to question.

Probability of employees greater than age of 44 = P(X>44)

P(X>44) = 1-P(X≤44)

Z=(X-44) = (X-38)/6

Thus, the question can be answered by using the normal table to find.

P(X≤44) = P(Z≤(44-38)/6)

= P(Z≤1)

= 84.1345%

Probability that the employees will be greater than age of 44 is, 100-84.1345 = 15.86%

So, the probability of the number of employees b/w 38&44 year’s of age

= P(x<44)-0.5 = 84.1345 - 0.5

= 34.1345%

Therefore, the statement that “more employees at the processing center are older than 44 than b/w 38 and 44” is true.

1. Probability of employees less than age of

30 = P(X<30)

Z = (X – 36) = (30-38)/6

Thus, the question can be answered by using the normal table to find,

P(X≤30) = P(Z≤(30-38)/6

= P(≤-1.333)

= 9.12%

So, the number of employees with probability 0.912 of them being under age.

30 = 0.0912\*400 = 36.48 (or 36 employees), therefore the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Solution: -

As we know that if X1~N(µ1,σ1^2) and X2~N(µ2,σ2^2) are two independent random variables then, X1+X2~N(µ1+µ2, σ1^2+σ2^2), and

X1-X2~N(µ1-µ2, σ1^2+σ2^2),

Similarly, if Z = ax+by, where X1 and X2 are as define above i.e., Z is linear

Combination of X1 and X2. Then,

Z~N(aµ1+bµ2, a^2σ1^2+b^2σ2^2)

Therefore, in the question

2X1~N(2µ, 4σ^2) and

X1+X2~N(µ+µ, σ^2+σ^2)~N(2µ,2σ^2)

2X1-(X1+X2) = N(4µ, 6σ^2).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Solution: -

Since we need to find out the values a and b, which are symmetric about the mean, such that the probability of random variable taking a value b/w them is 0.99, we have to work out in reserve order,

The probability of getting value b/w a&b value should be 0.99

So the probability of going wrong (or) the probability outside the a and b are a is 0.01,

(i.e., 1 – 0.99).

The probability towards right from b = +0.05(i.e., 0.01/2)

So since we’ve the probabilities of a and b which has got these probabilities.

By finding the standard normal variable Z(Z-table),

We can calculate the X-values.

Z = (X-µ)/σ for probability 0.05 the Z-value is -2.57(from z-table).

Z\*σ+µ=X

Z(-0.005)\*20+100 = -(-2.57) \* 20+100 = 151.4

Z(+0.005)\*20+100 = (-2.57) \* 20+100 = 48.6

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Solution: -

1. P(P<P) = 95% = P(P-12/ < P-12/) = 0.95

Therefore, P-12/~N(0,1)i.e., (P-12) = 0.95

Here = CDF of standard normal random variable

P-12/) = 0.95 => [P-12/] = 1.644854

P => 12+1.644354\* [From standard normal distribution]

= 13.41

$13.41 = 45\*13.41 => Rs603.68

1. Assume the 5th percentile of profit to be Z

Then, p(p<q) = 5% = P(P-12/) = 0.05

From standard normal distribution,

P-12/) = 0.05 => P-12/ = -1.644854

Therefore, P = 12-1.644854\*

= 10.585

Therefore, P = $10.54 = Rs476.33

1. For first distribution,

Probability of making a loss = Probability of profit less than zero

Probability of profit less than zero = P(Profit 1-5/<0-5/)

≈P(Profit 1-5/ < 0.88388)

≈0.18838

(This value is gotten from standard normal distribution table)

For 2nd division,

Probability of making a loss => probability of profit 2 less than zero

Profit Z~N(7,42) = Profit-7/~N(0,1)

Probability of 2<0) = P(Profit 2-7/~N(0,1)

Probability (Profit2<0) = P(Profit 2-7/ <-1.08012348)

≈0.140044

(Gotten from standard normal distribution table.)